National Aeronautics and Space Administration

Exploring Space Through ALGEBRA

Communications and the Lunar Outpost

Instructional Objectives
Students will
- use algebraic properties to solve equations for a given variable;
- apply formulas to calculate various geometric measures; and
- analyze data to determine a solution to a real life problem.

Prerequisites

Prior to this activity, students should have had experiences applying formulas. Students should be familiar with using calculators in evaluating formulas and have a basic knowledge of circles, trigonometric functions, and point of tangency.

Background

This problem is part of a series of problems that apply Algebra and Geometry principles to U.S. Space Exploration policy.

Exploration provides the foundation of our knowledge, technology, resources, and inspiration. It seeks answers to fundamental questions about our existence, responds to recent discoveries and puts in place revolutionary techniques and capabilities to inspire our nation, the world, and the next generation. Through NASA, we touch the unknown, we learn and we understand. As we take our first steps toward sustaining a human presence in the solar system, we can look forward to far-off visions of the past becoming realities of the future.

The vision for space exploration includes returning the space shuttle safely to flight, completing the International Space Station, developing a new exploration vehicle and all the systems needed for embarking on extended missions to the Moon, Mars, and beyond.

By 2020, NASA astronauts will again explore the surface of the Moon. This time, we're going to stay, building outposts and paving the way for eventual journeys to Mars and beyond.

The crewed and robotic return to the Moon requires robust and reliable communications. It will be important to maintain constant communications with Earth. Therefore, 24 hours per day/7 days per week coverage at the outpost is a requirement. This will likely be accomplished by a combination of communication satellites in orbit around the Moon and communication equipment on the lunar surface.

Grade Level
9 -12

Subject Area
Mathematics: Geometry, Algebra II

Key Concept
Pythagorean Theorem, arc length, inverse tangent

Teacher Prep Time
15-20 minutes

Problem Duration
60-90 minutes

Technology
TI-83 Plus Family; TI-84 Plus family; TI Explorer; other scientific calculator

Materials
- Student Edition

Degree of Difficulty
Moderate to Difficult

Skill
Calculator use, solving problems in a geometric context; operations with trigonometric functions; and use of formulas

----

NCTM Principles and Standards
- Number and Operations
- Algebra
- Geometry
- Measurement
- Problem Solving
- Communication
- Connections
- Representation

The lunar habitat (Figure 1) on the Moon's surface will need video downlink capability to Earth. In addition to the communication requirements between the lunar surface and Earth, it will also be important to maintain constant communications between surface crew members, regardless of their distance from the outpost.
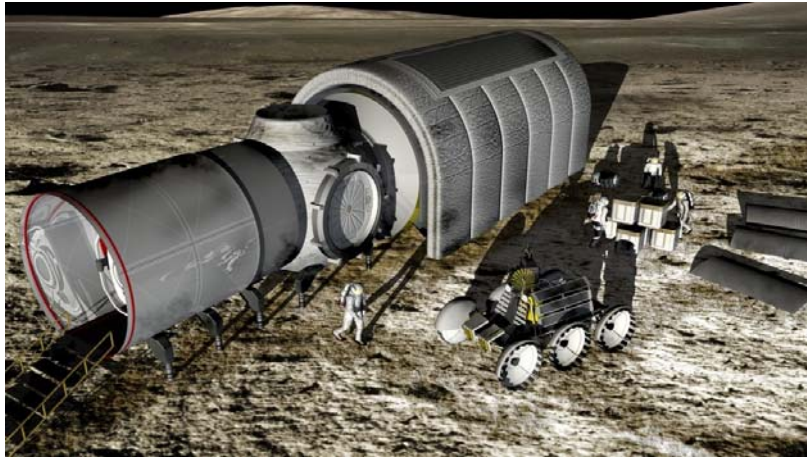

Figure 1: NASA concept of a Lunar habitat, airlock, and vehicles (not to scale)

Surface to surface communications involves communicating between astronauts, rovers, robots, habitats, power stations, and science experiments, as well as communication within the habitats. For surface-based communication systems, there is a line of sight limitation on rover communication with the habitat. Astronauts must have either the habitat or the rover in their line of sight to maintain communications with Earth.

The communications system should be easily expandable. Future missions will not want to abandon existing equipment, but instead incorporate existing equipment into an expanding lunar communications system.

These plans give NASA a huge head start in getting to Mars. We will already have rockets capable of transporting heavy cargo as well as a versatile crew capsule. A lunar outpost just three days away from Earth will give us needed practice of "living off the land" away from our home planet, before making the longer trek to Mars.

For more information about lunar outposts, communications and the U.S. Space Exploration policy, visit www.nasa.gov.

NCTM Principles and Standards

Algebra
- Identify essential quantitative relationships in a situation and determine the class or classes of functions that might model the relationships.
- Model and solve contextualized problems using various representations, such as graphs, tables, and equations.

Geometry
- Recognize and apply geometric ideas and relationships in areas outside the mathematics classroom, such as art, science, and everyday life.
- Use trigonometric relationships to determine lengths and angle measures.
- Use geometric models to gain insights into, and answer questions in, other areas of mathematics.

Measurement
- Understand both metric and customary systems of measurement.
- Understand relationships among units and convert from one unit to another within the same system.
- Select and apply techniques and tools to accurately find length, area, volume, and angle measures to appropriate levels of precision.

Problem Solving
- Solve problems that arise in mathematics and in other contexts.

Communication
- Organize and consolidate their mathematical thinking through communication.
- Communicate their mathematical thinking coherently and clearly to peers, teachers, and others.
- Use the language of mathematics to express mathematical ideas precisely.

Connections
- Recognize and apply mathematics in contexts outside of mathematics.

Representation
- Use representations to model and interpret physical, social, and mathematical phenomena.

Problem

Suppose a lunar outpost has been established on the rim of Shackleton crater. Shackleton crater is small, about 19 kilometers in diameter, and is not in range of any satellite. The rim is slightly raised and in an area of near-permanent sunlight that provides access to solar power. The crater's permanently dark interior is a cold trap that may contain water ice. This site is within the South Pole-Aitken (SPA) basin, the oldest and biggest impact feature on the Moon. The SPA basin is about 12 kilometers deep and exploration of its geologic features may provide useful information about the lunar interior.

Whenever astronauts travel away from the lunar outpost for scientific study or exploration, constant communications will be provided by surface to surface communication towers. Receivers within a certain radius of the communication tower antenna can pick up signals. Because the Moon is a sphere, the surface to surface signals cannot be received beyond the point of tangency, P, of the line of sight distance, *d* (Figure 2).

Precision in these calculations will be critical to the communication signal, therefore carry all calculations to two decimal places. Also, if you are using a calculator, make sure it is set to degrees (not radians).

Problem Diagrams

h = tower height

r = radius of the Moon (1738.14 km)

d = line of sight distance

P = point of tangency

a = arc length

θ = central angle measure
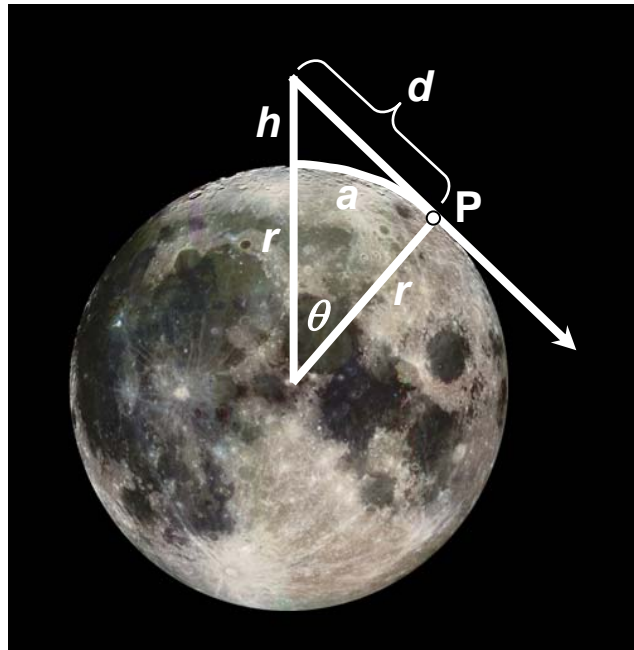
C = circumference

Figure 2: Problem Diagram
NOTE: Diagram is exaggerated to show relationship and reference points.
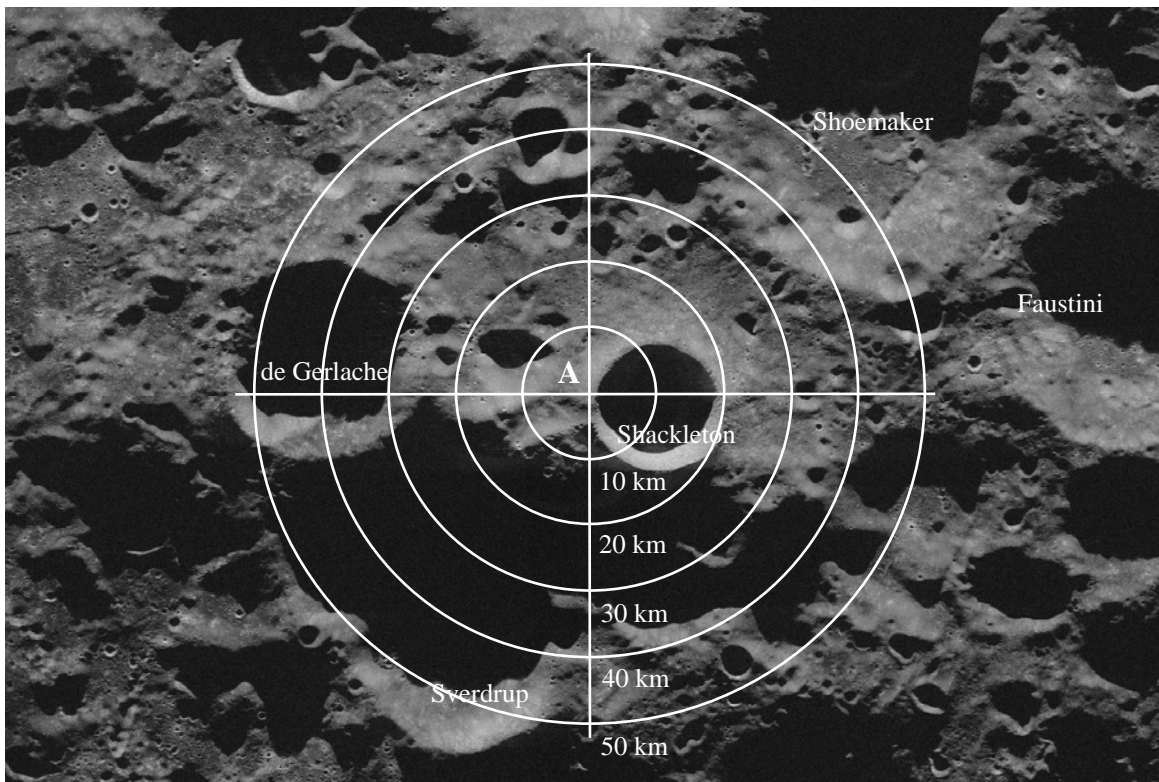

Figure 3: Lunar south pole region
(Cornell University/Smithsonian Institution Image)

Lesson Development

This activity focuses on the relationship between the height of a communication tower and the tower's communications range. Students will use formulas to calculate the line of sight distance and use problem-solving skills to determine relationships between line of sight distance, central angle measure, and arc length.

Students will work in groups of four recording their work in the tables provided. Encourage students to perform the calculations for the height of each tower and verify their answers with other members of their group. The teacher may need to review conversions between kilometers and meters because the tower heights are given in meters and crater measurements are given in kilometers. Students will design a plan to provide communication to all five proposed sites based upon the constraints they discover while solving the initial problems.

Wrap-Up

Once the exercise is completed, have groups share their design plans. Groups should discuss and evaluate the various approaches taken and the methods used to create each plan, as well as careers, skills, and expertise of personnel needed to implement their plan.

Extensions

Have students share their communication design proposal with the class, as if they were presenting to NASA. Groups might choose to use three-dimensional models, power point presentations, a brochure, podcast, movie, etc.

Solution Key

Suppose a lunar outpost has been established on the rim of Shackleton crater. Shackleton crater is small, about 19 kilometers in diameter, and is not in range of any satellite. The rim is slightly raised and in an area of near-permanent sunlight that provides access to solar power. The crater's permanently dark interior is a cold trap that may contain water ice. This site is within the South Pole-Aitken (SPA) basin, the oldest and biggest impact feature on the Moon. The SPA basin is about 12 kilometers deep and exploration of its geologic features may provide useful information about the lunar interior.

Whenever astronauts travel away from the lunar outpost for scientific study or exploration, constant communications will be provided by surface to surface communication towers. Receivers within a certain radius of the communication tower antenna can pick up signals. Because the Moon is a sphere, the surface to surface signals cannot be received beyond the point of tangency, P, of the line of sight distance, *d* (Figure 2).

Precision in these calculations will be critical to the communication signal, therefore carry all calculations to two decimal places. Also, if you are using a calculator, make sure it is set to degrees (not radians).

1. Astronauts are traveling away from the outpost to study some of the surrounding craters.

    a. If the height of a communication tower at the lunar outpost (Point A in Figure 3) is 50 meters above the lunar surface, how far away can they explore and still be within communication range?

Step 1: Using Figure 2 where h is the tower height, r is the radius of the Moon (r = 1738.14 km), and d is the line of sight distance, determine the line of sight distance using the Pythagorean Theorem.

$$r^2 + d^2 = (r+h)^2$$

$$d = \sqrt{(r+h)^2 - r^2}$$

$$d = \sqrt{(1738.14 \text{ km} + 0.05 \text{ km})^2 - (1738.14 \text{ km})^2}$$

$$d = 13.18 \text{ km}$$

Step 2: Using Figure 3 where θ is the central angle measure and a is the arc length, determine the arc length (maximum distance the astronauts can explore within communication range).

$$\theta = \tan^{-1}\left(\frac{d}{r}\right)$$

$$\theta = \tan^{-1}\left(\frac{13.18 \text{ km}}{1738.14 \text{ km}}\right)$$

$$\theta = 0.43°$$

$$a = \frac{\theta}{360°} \cdot C$$

$$a = \frac{\theta}{360°} \cdot 2\pi r$$

$$a = \frac{0.43°}{360°} \cdot 2\pi(1738.14 \text{ km})$$

$$a = 13.04 \text{ km}$$

b. Using Table 1, what craters will the astronauts be able to explore?

Table 1: Distance from Outpost to Center of Crater Site

Proposed Site	Distance from Outpost
Shackleton crater	~ 10 km
de Gerlache crater	~ 40 km
Sverdrup crater	~ 45 km
Shoemaker crater	~ 70 km
Faustini crater	~ 80 km

Shackleton crater

2. In order to increase the line of sight distance, a higher communication tower must be built. Using what you have learned in question 1, complete Table 2.

Table 2: Various Tower Heights and Related Measurements

Tower Height, h (m)	Line of Sight Distance, d (km)	Central Angle Measure, θ (deg)	Arc Length, a (km)	Line of Sight Distance per Meter of Tower Height
50	13.18	0.43	13.04	0.26
100	18.65	0.61	18.51	0.19
200	26.37	0.87	26.39	0.13
400	37.29	1.23	37.31	0.09
800	52.74	1.74	52.79	0.07

a. Is there a relationship between line of sight distance and arc length?

   They appear to be approximately equal. The height of the tower in relation to the radius of the Moon is so small that the difference between the line of sight distance and arc length is minimal.

b. Is there a relationship between the tower height and central angle measure? Explain your answer.

   Yes. As the tower height increases, the central angle measure increases.

c. What tower height gives the maximum line of sight distance per meter of tower height?

   The 50 meter tower gives the maximum line of sight distance per meter of tower.

d. As the tower height increases does the line of sight distance per meter of tower height increase or decrease?

   It decreases.

e. As the height of the communication tower doubles in size, does the corresponding line of sight distance double? Why or why not?

   No. The height of the tower in relation to the radius of the Moon is so small doubling the height of the tower contributes very little to increasing the line of sight distance.

f. An 800-meter tower would provide communication to de Gerlache and Sverdrup craters. Approximately how tall would a tower need to be in order to provide communication to Faustini crater? Is either of these heights practical for a communication tower on the Moon? Explain your reasoning.

$$(r + h)^2 = r^2 + d^2$$

$$r + h = \sqrt{r^2 + d^2}$$

$$h = \sqrt{r^2 + d^2} - r$$

$$h = \sqrt{\left((1738.14\ \text{km})^2 + (80\ \text{km})^2\right)} - 1738.14\ \text{km}$$

$$h = 1.84\ \text{km or 1840 m}$$

Neither an 1840 m tower nor an 800 m tower is practical. Students may find it easier to understand the height by converting to feet. After doing the conversions, students will find 1840 m ≈ 6,037 ft ≈ 1.1 miles and 800 m ≈ 2,625 feet ≈ 0.5 mile.

Note to teacher: The average cell phone tower on Earth is between 45 and 80 meters. The tallest manmade structure on Earth is a communication tower that is 628 meters high.

3. Given what you have discovered in questions 1 and 2, design a plan to provide communication to all five proposed crater sites. Include the type of professions (skills and expertise) you would need in order to successfully implement your plan. Also, make sure to review Figure 3 and decide what lunar features will need to be accounted for in your plan.

   Answers may vary. Student's plans might include the height of the proposed tower, as well as the feasibility of different locations on the surface of the Moon for optimal communication with all five crater sites. Skills and expertise needed could be supplied by professions such as mechanical and electrical engineers, communication specialists, metal workers, and carpenters.

Contributors

Thanks to the subject matter experts for their contributions in developing this problem:

NASA Experts

NASA Johnson Space Center

John   Gruener
Flight   Systems Engineer
        Lunar Surface System Project Office
        Constellation Program

Problem Development

NASA Langley Research Center

Chris Giersch
Communications and Education Lead
Exploration and Flight Projects Directorate

Sharon Bowers
Educator in Residence
Office of Strategic Communications and Education
NASA Langley Research Center

NASA Johnson Space Center

Traci Knight
Graphics Specialist
Human Research Program Education and Outreach

Monica Trevathan
Education Specialist
Human Research Program Education and Outreach

Natalee Lloyd
Educator, Secondary Mathematics
Human Research Program Education and Outreach

Virginia Beach City Public Schools

Josephine Letts
Mathematics Teacher
Ocean Lakes High School

Diana Hirschi
Mathematics Teacher
Ocean Lakes High School

University of North Carolina Charlotte

Dr. David Pugalee
Research Associate
Center for Mathematics, Science & Technology Education

Exploring Space through Algebra

Communications and the Lunar Outpost

Feedback Form

Please take a minute to complete this feedback form. Your input will help improve this product and will help us create new, useful material.

Fax the completed form to: (281) 461-9350 – Attention: Martha Grigsby

Or type your responses in an email and send to: Martha.Grigsby@Tietronix.com

Please circle the appropriate response.

1. This problem was useful in my classroom.   YES   NO

2. The problem successfully accomplished the stated Instructional Objectives.   YES   NO

3. I will use this problem again.   YES   NO

4. Please provide suggestions for improvement of this problem and associated material:

   _____

   _____

   _____

5. Please provide suggestions for future Algebra problems, based on NASA topics, that you would like to see developed:

   _____

   _____

   _____

   Thank you for your participation.

   Please fax this completed form to Martha Grigsby at (281) 461-9350.